Engineering Mechanics

Mehanics is a branch of Science which deals about body is in rest Condition or motion under the action of forces.

- Engineering Mechanics
  - Statics
  - Dynamics
    - Kinematics
    - Kinetics

⇒ **Statics :-**

Statics is a branch of mechanics which deals about Rest Condition of the body under the action of force.

⇒ **Dynamics :-**

Dynamics is a branch of mechanics which deals about motion of the body.

⇒ **Kinematics :-**

In which the description of motion of body independent of causes of motion.

⇒ **Kinetics :-**

In which both motion and its causes are Considered.

Basic definitions :-

1. **Matter :-** The matter is a substance which occupies space, possesses mass and offers resistance to any External force. Ex:- Iron, Stone, wood etc.

2. **particle :-** It is an object that has infinitely small volume but has a mass which can be considered to be concentrated at a point.

3. **Body :-** body has a definite shape and Consists of numbers of particles.

There are two type of body.

(i) Elastic body (ii) plastic body.

(i) **Elastic body :-** Body undergoes deformation but regains its original shape after removal of the external force.

(ii) **Plastic body :-** Body undergoes deformation but donot regains its original shape after removal of the external force.

4. **Space :-** Space is a region which extends in all directions and Contains every thing in it.

5. **Time :-** Time is a measure of succession of events. The unit of time

6. **Motion :-** when a body changes its position with respect to other bodies. then body is said to be in motion.

Trajectory:- Trajectory is path followed by a body during its motion. It may be a straight line or a curve.

Mass:- The quantity of matter present within the System. is called mass.

Weight:- weight is a force which the system exerts due to gravitational acceleration.

Force:- Force is an external agent which tends to change or change the state of the body.

Characteristics of a force are.

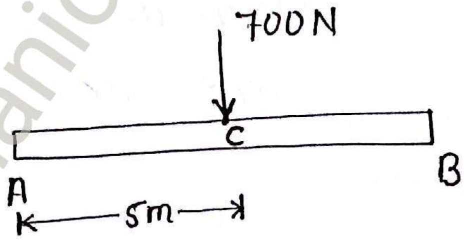
(a) Its magnitude.

(b) Its point of application.

(c) Its direction.

(d) Line of action.

EX:-



→ Magnitude is 700 N

→ The point of application is at a distance of 5 m from A point.

→ The line of action of force is vertical.

→ The direction is in downward.

⇒ The unit of force is Newtons (N) and it is a vector quantity.

System of forces:-

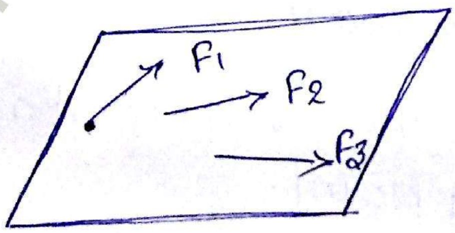
when several forces of different magnitude and direction act upon a body, than it is called force of system or system of forces.

System of forces

- Coplanar forces
  - Concurrent
  - parallel
    - Like
    - unlike
  - Non-Concurrent Non parallel
- Non coplanar forces
  - Concurrent
  - parallel
  - Non concurrent Non-parallel

Collinear forces:- The Line of action of all forces lie along the same straight line.

Coplanar forces:- When more than one forces acting in a single plane, than thea forces are called in Coplanar.



Concurrent forces:- When more than one forces acting at a single point, than the forces are called concurrent forces.

Non-Coplanar forces:- When more than one forces acting in different plane is called Non-coplanar forces.

Equilibrium:- When two or more than two forces act on a body in such a way that the body remains in a state of rest or of uniform motion, then the system of forces is said to be in equilibrium.

Resultant:- When a body is acted upon by a system of forces then vectorial sum of all the forces is known as resultant. Hence resultant refers to the single force which produce the same effect as is done by the combined effect of several forces.

Fundamental principles of mechanics

The fundamental principle of mechanics are:-

1. Newton's law of motions.
2. Newton's law of gravitation.
3. parallelogram law.
4. principle of transmissibility.

Newton's law of motion:-

(1) Newton's first law of motion:-

Every body continues in its state of rest or of uniform motion in a straight line if

There is no unbalanced force acting upon it.

Newton's second law of motion:-

The rate of change of linear momentum is directly proportional to the impressed force and it takes place in the direction of the impressed force.

Newton's third law of motion

To every action, there is equal and opposite reaction.

⇒ Newton's law of gravitation:-

Every body in the universe attracts every other body with a force directly proportional to the product of their mass and inversely proportional to the square of the distance separating them.

$$F \propto m_1 m_2$$

$$\propto \frac{1}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

$$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

principle of transmissibility.

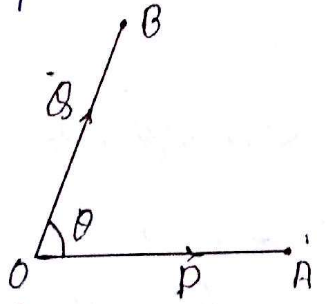
The Condition of equilibrium or motion of rigid body remains unchanged. If a force acting at a given point of the rigid body is replaced by a force of same magnitude and direction but action at a different point provided that the two forces have the same line of action.

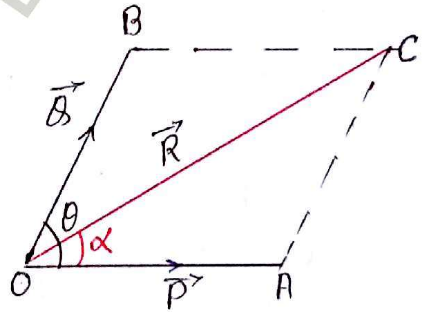
Law of forces:-

1) parallelogram law of forces.
2) Triangle law of forces.
3) Polygon law of forces.

1. Parallelogram law of forces:-

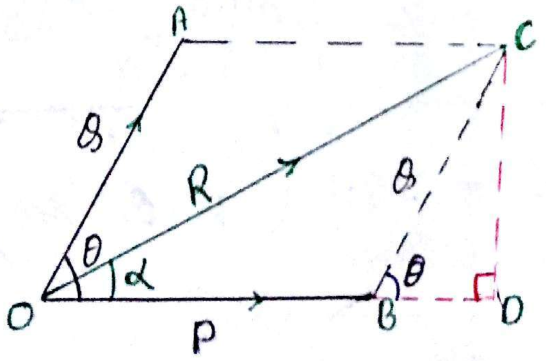
If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.





proof:-

Let us consider two forces P and Q acting on a body. The force P and Q represented in magnitude and direction by $\vec{OA}$ and $\vec{OB}$ respectively and angle between the force is θ. and the digonal $\vec{OC}$ Represent its resultant of the IIelogram OACB.



Drop perpendicular from 'C' and let it meet OA extend at point D.

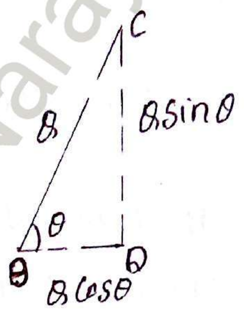
$\because \quad OA \parallel BC \quad \therefore \quad OA = BC = Q.$

from ΔBCD

$$\sin\theta = \frac{CD}{BC} \quad \& \quad \cos\theta = \frac{BD}{BC}$$

$$\Rightarrow \sin\theta = \frac{CD}{Q} \qquad \Rightarrow \cos\theta = \frac{BD}{Q}$$

$$\Rightarrow CD = Q\sin\theta \qquad \Rightarrow BD = Q\cos\theta.$$



from :- ΔOCD

$$OC^2 = OD^2 + CD^2$$

$$\Rightarrow OC^2 = (OA + AD)^2 + CD^2$$

$$\Rightarrow OC^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$\Rightarrow R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$

$$\Rightarrow R^2 = P^2 + 2PQ\cos\theta + Q^2(\sin^2\theta + \cos^2\theta)$$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\theta \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$ Magnitude

For direction.

In ΔOCD

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

$$= \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)}$$

Special cases:-

Case 1. When the two forces act $\theta = 90^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$$= \sqrt{P^2 + Q^2 + 2PQ \times 0} \quad [\because \cos 90^\circ = 0]$$

$$\boxed{R = \sqrt{P^2 + Q^2}}$$

and

$$\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{Q\sin 90^\circ}{P + Q\cos 90^\circ}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$$

Case 2. When the two forces act in the same line

$$\theta = 0^\circ$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ}$$

$$= \sqrt{P^2 + Q^2 + 2PQ} \quad [\because \cos 0^\circ = 1]$$

$$R = \sqrt{(P+Q)^2}$$

$$\boxed{R = P + Q}$$ maximum magnitude.

Case 3. When the $\theta = 180^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ}$$

$$= \sqrt{P^2 + Q^2 - 2PQ} \quad [\because \cos 180^\circ = -1]$$

$$= \sqrt{(P-Q)^2}$$

$$\boxed{R = P - Q}$$ Minimum magnitude.

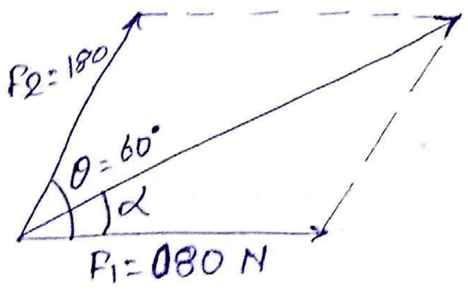
Q.1. Two forces of magnitude 80N and 160 N are acting simultaneously at a point. The angle between the forces are 60°. Find the magnitude and direction of resultant force acting on the point.

Soln:- Given, $F_1 = 80\ N$ and $\theta = 60°$

$F_2 = 160\ N$

To find $R = ?$

$\alpha = ?$



we know that

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(80)^2 + (160)^2 + 2\cdot 80\cdot 160\cdot \cos 60°}$$

$$= \sqrt{6400 + 25600 + 25600 \times \frac{1}{2}}$$

$$= \sqrt{44800}$$

$$\boxed{\therefore R = 211.660\ N}$$

for direction.

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

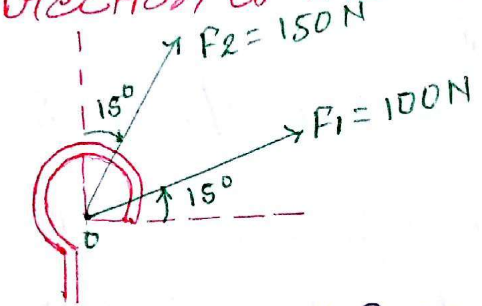
$$= \frac{180 \cdot \sin 60°}{80 + 180\cos 60}$$

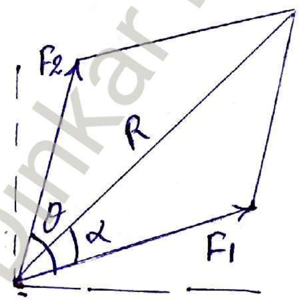
$$= 0.866$$

$$\alpha = \tan^{-1}(0.866)$$

$$= 40.9 \text{ degree with force } F_1$$

Q.2. An eye bolt as shown in figure below is subjected to two force $F_1 = 100N$ and $F_2 = 150N$ Determine the magnitude and direction of the resultant force.





Soln angle b/w F_1 and F_2 is

$$\theta = [90^\circ - 15^\circ - 15^\circ] = 60^\circ$$

$$F_1 = 100\ N,\quad F_2 = 150N$$

$$\because\ R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2\times100\times150\cos60^\circ}$$

$$= \sqrt{47500}\ N$$

$$\boxed{R = 217.97 = 218N}$$

For: Direction:

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

$$= \frac{150\cdot\sin60}{100 + 150\cdot\cos60^\circ}$$

$$\tan\alpha = 0.742$$

$$\alpha = \tan^{-1}(0.742)$$

$$\boxed{\alpha = 36.575^\circ}$$ with force F_1

Q.3. The resultant of two forces 'P' and Q acting at a point is 'R'. If Q is doubles force R also get doubled and if Q is reversed R is again doubled. Show that the ratio of P, Q and R is given by.

$$P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$$

Soln :- From parallelogram law of forces

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta \quad \text{—— (I)}$$

Case 1:- when Q is doubled, R also get doubled.

$$\therefore \quad (2R)^2 = P^2 + (2Q)^2 + 2P(2Q)\cos\theta$$

$$\Rightarrow \quad 4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta \quad \text{—— (II)}$$

Case 2:- when Q reversed in direction, R is again doubled.

$$\therefore \quad (2R)^2 = P^2 + (-Q)^2 + 2P(-Q)\cdot\cos\theta$$

$$4R^2 = P^2 + Q^2 - 2PQ\cos\theta \quad \text{—— (III)}$$

Adding eqn (I) and (III)

$$R^2 = P^2 + Q^2 + \cancel{2PQ\cos\theta}$$
$$4R^2 = P^2 + Q^2 - \cancel{2PQ\cos\theta}$$
$$5R^2 = 2P^2 + 2Q^2 \quad \text{—— (IV)}$$

and

eqn (III) × 2 + eqn (II)

$$8R^2 = 2P^2 + 2Q^2 - \cancel{4PQ\cos\theta}$$
$$4R^2 = P^2 + 4Q^2 + \cancel{4PQ\cos\theta}$$
$$12R^2 = 3P^2 + 6Q^2$$
$$4R^2 = P^2 + 2Q^2 \quad \text{—— (V)}$$

again eqn (IV) − (V)

$$5R^2 = 2P^2 + \cancel{2Q^2}$$
$$\underline{-4R^2 = -P^2 \pm \cancel{2Q^2}}$$
$$R^2 = P^2$$

$$\boxed{\therefore\ R = P} \quad \text{— (VI)}$$

Put the value of R is eqn (V)

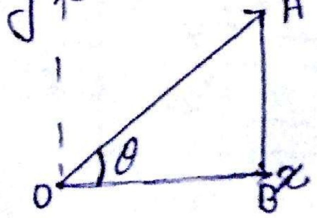
$$4P^2 = P^2 + 2Q^2$$
$$\Rightarrow 3P^2 = 2Q^2$$
$$\Rightarrow Q^2 = 3/2\, P^2$$
$$\boxed{Q = \frac{\sqrt{3}}{\sqrt{2}} P}$$

$$\therefore P : Q : R = P : \frac{\sqrt{3}}{\sqrt{2}} P : P$$

$$\boxed{\therefore\ P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}}$$ proved

Resolution of forces:—

When a force acting at a point with θ angle from the horizontal than It resolve in ~~two~~ two Component verticle or Horizontal.



From △OAB

$$\sin\theta = \frac{AB}{OA}$$
$$\Rightarrow \sin\theta = \frac{AB}{F}$$
$$\boxed{AB = F\sin\theta}$$

and $\cos\theta = \frac{OB}{OA}$

$$\Rightarrow \cos\theta = \frac{OB}{F}$$

$$\boxed{OB = F\cos\theta}$$

It means the Force resolve into two part. $F\cos\theta$ and $F\sin\theta$.

Resultant of Coplanar - Concurrent forces:-

Steps:-

(i) Find the Component of each force in the system in two mutually perpendicular x and y direction.

(ii) Make algebric addition of Components in each direction to get two Components ΣF_x and ΣF_y

(iii) obtain the resultant both in magnitude and direction by two Component.

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$ magnitude

$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}}$$ Direction

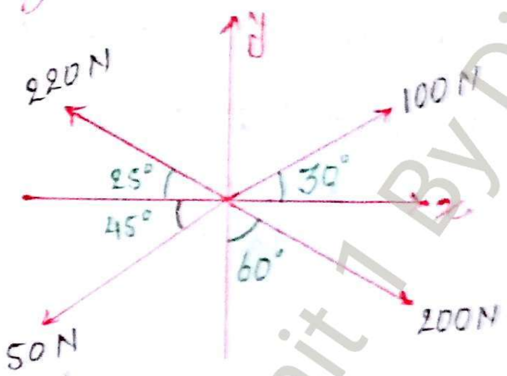
For Direction:-

$$\tan\alpha = \frac{\Sigma y}{\Sigma x} = \frac{50.67}{194.6} = 0.260$$

$$\alpha = \tan^{-1}(0.260)$$

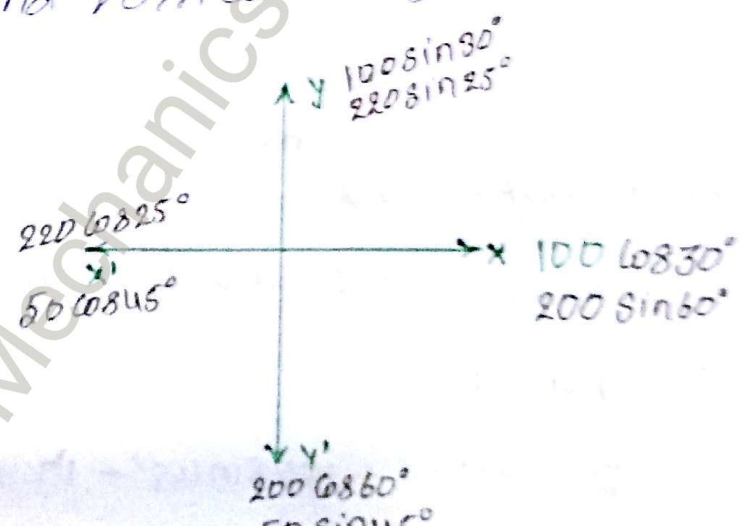
$$\boxed{\alpha = 14.57} \quad \text{Ans}$$

Q. A System of four forces acting on a body is shown in fig. Determine the resultent.



Soln

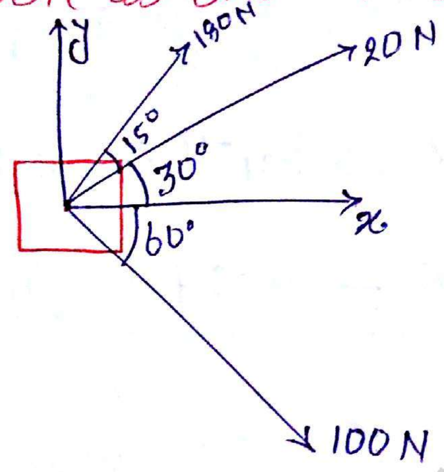
Composition of forces is represent on horizontal and vertical axis.



$$\Sigma x = 100\cos 30° + 200\sin 60° - 220\cos 25° - 50\cos 45°$$
$$= 25.06 \text{ N.}$$

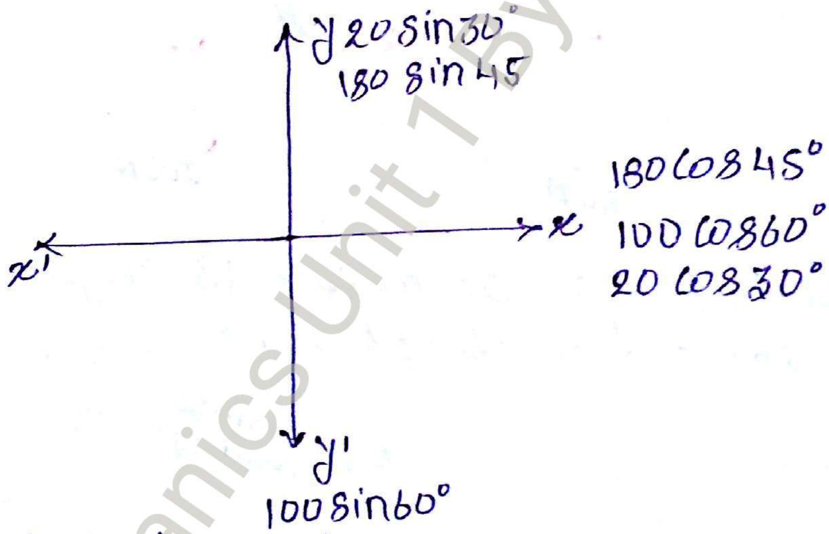
$$\Sigma y = 100\sin 30° + 220\sin 25° - 200\cos 60° - 50\sin 45°$$
$$= 7.62 \text{ N.}$$

Q. Determine the resultant of the three forces acting on a block as shown as.



Soln :-

Component of all the forces are represented on the vertical and Horizontal axis.



Horizontal Component

$$\Sigma x = 100\cos 60 + 20\cos 30 + 180\cos 45^\circ$$

$$= 194.6\ N$$

$$\Sigma y = 20\sin 30^\circ + 180\sin 45^\circ - 100\sin 60^\circ$$

$$50.67\ N$$

Resultent :-

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(194.6)^2 + (50.67)^2}$$

$$\boxed{R = 201.08}$$

For Resultent :-

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(25.06)^2 + (7.62)^2}$$

$$\boxed{R = 26.193\ N} \quad \text{Ans}$$

For Direction

$$\tan\alpha = \frac{\Sigma y}{\Sigma x} = \frac{7.62}{25.06}$$

$$\alpha = \tan^{-1}\left(\frac{7.62}{25.06}\right) = 16.913$$

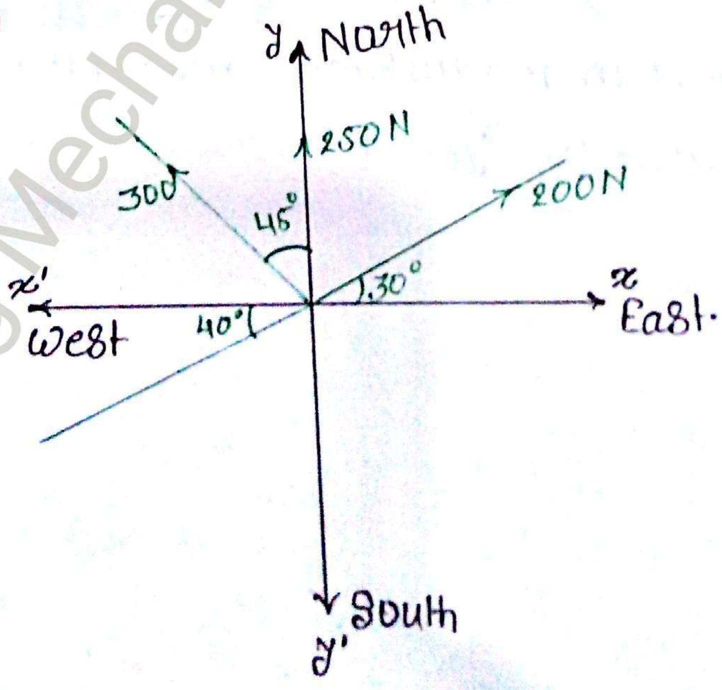
$$\boxed{\alpha = 16.913} \quad \text{Ans}$$

Q. Determine the magnitude and direction of the resultant of the following set of forces acting on a body.

1. 200 N inclined 30° with east towards north.
2. 250 N towards the north.
3. 300 N towards north west and
4. 350 N inclined at 40° with west towards south.

What will be the equilibrant of the given force system.?

Soln



Resolving all the forces along x-direction and y-direction.

$$\Sigma F_x = 200\cos 30^\circ + 250\cos 90^\circ + 300\cos 45^\circ - 350\cos 40^\circ$$

$$= -307 \text{ (along } Ox'\text{) N}$$

$$\Sigma F_y = 200\sin 30^\circ + 250\sin 90^\circ + 300\sin 45^\circ - 350\sin 40^\circ$$

$$= 337.4 \text{ N (along } Oy\text{)}$$

Resultent

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(-307)^2 + (337.4)^2} = 456 \text{ N}$$

Direction

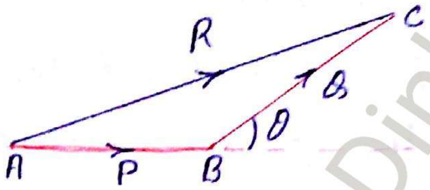
$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{337.4}{-307}\right) = -47.70^\circ$$

than the equilibrant of this system is 456 N in magnitude and 47.7° to the x-axis in forth quadrant.

Triangle law of forces.

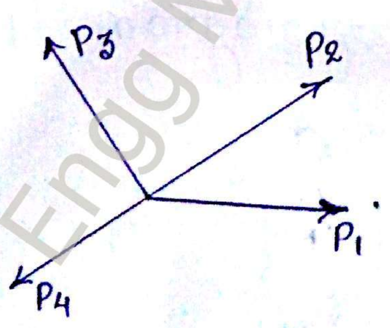
"If two forces acting on a body are represented by the sides of a traingle taken in order, their resultant is represented by the closing side of the traingle taken in the opposite order"



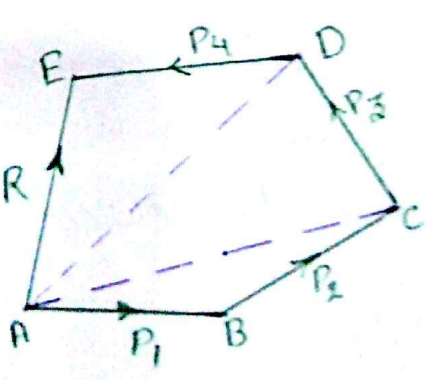
$$\boxed{\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}}$$

Polygon law of forces

If a number of concurrent forces acting on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then the resultant is represented in magnitude and direction by closing side of the polygon, taken in opposite order.



(a)



(b)

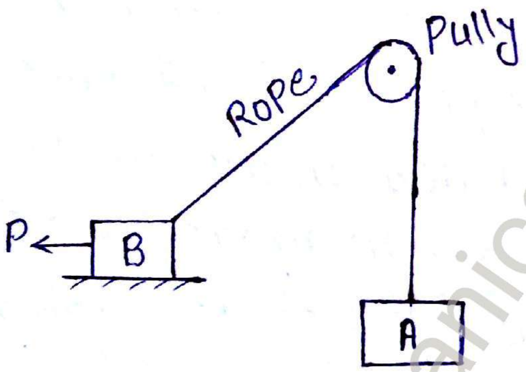
Free body diagrame:-

A free body diagram is a sketch of an object which is free from all the Contact surface and all the forces acting on it are drawn.

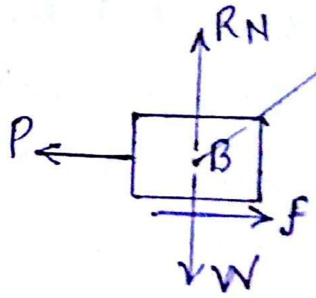
Asumpsions for free body diagram.

1. Select the system.
2. All the surface Contect is removed.
3. Draw all the forces on the system.
4. Extra force is introduce wt (W) acting it self.

Ex:-

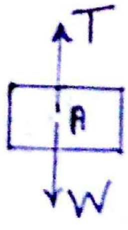


Free body diagram of B



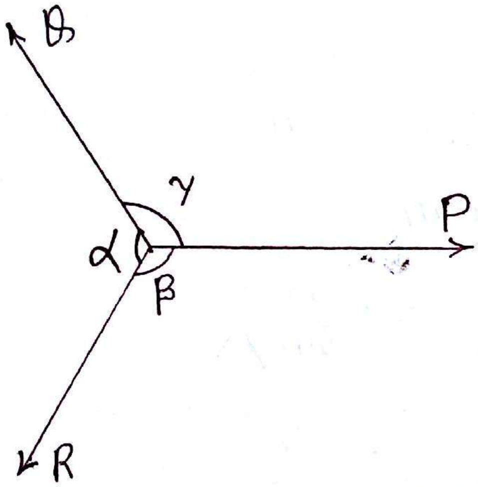
W = wt.
f = Friction force.
T = Tension.
R_N = Normal Rection

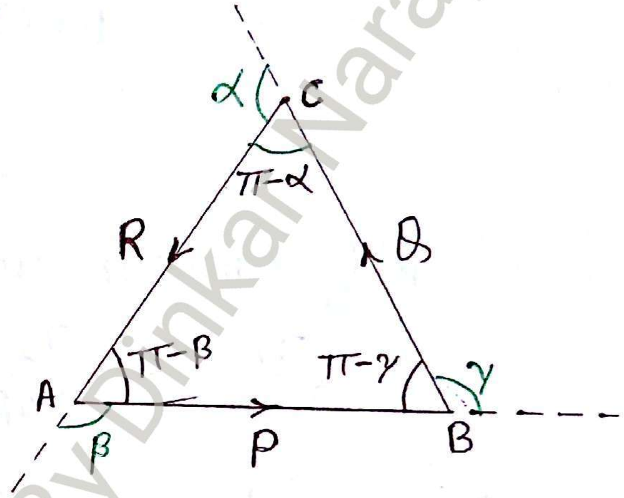
Free body diagram of A



Lami's theorem

"If a body is in equilibrium under the action of three forces, then each force is proportional to the sine of the angle between the other two forces"



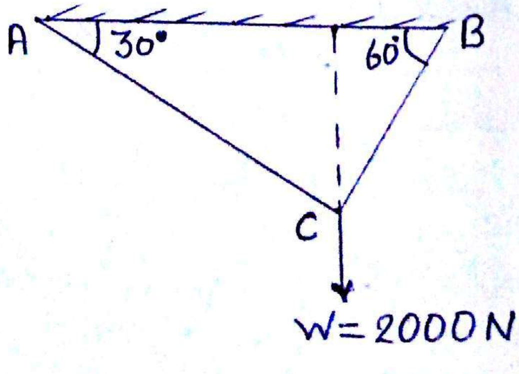


Applying sine rule for the traingle ABC

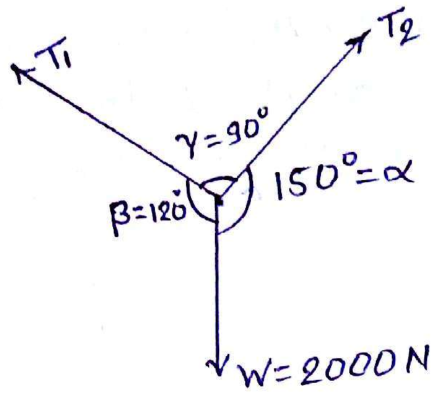
$$\frac{AB}{\sin(\pi-\alpha)} = \frac{BC}{\sin(\pi-\beta)} = \frac{CA}{\sin(\pi-\gamma)}$$

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

Q. A weight of 2000 N is supported by two chains AC and BC as shown in fig. Determine the tension in each chain.



Soln :- Free body of point 'c'



Aply Lami's theorem

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\gamma}$$

or
$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{W}{\sin 90^\circ}$$

$\therefore \frac{T_1}{\sin 150} = \frac{W}{\sin 90}$ or $\frac{T_2}{\sin 120} = \frac{W}{\sin 90}$

$\Rightarrow \frac{T_1}{\sin 150} = \frac{2000}{\sin 90}$ $\Rightarrow \frac{T_2}{\sin 120} = \frac{2000}{\sin 90}$

$\Rightarrow T_1 = \frac{2000}{\sin 90} \times \sin 150$ $\Rightarrow T_2 = \frac{2000}{\sin 90} \times \sin 120$

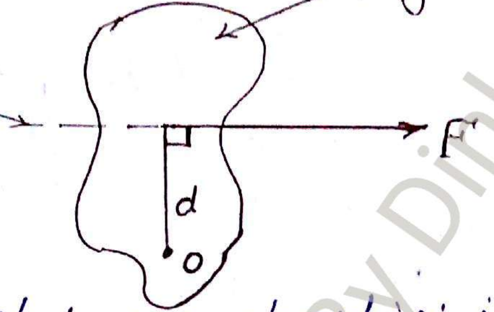
$\Rightarrow T_1 = \frac{\cancel{2000}}{1} \times \frac{1}{\cancel{2}}$ $\Rightarrow T_2 = \frac{2000}{1} \times \sin 120$

$\boxed{T_1 = 1000\text{ N}}$ $\boxed{T_2 = 1732\text{ N}}$

Moment of force and parallel forces:-

Moment of a force:-

Moment of force about a point is defined as the turning tendency of force about that point. It is measured by the product of force and the ⊥lar distance of the line of action of the force from that point.



The moment of force about 'O' is

$$\boxed{M_O = F \cdot d}$$

When force acting on a body has two effects.

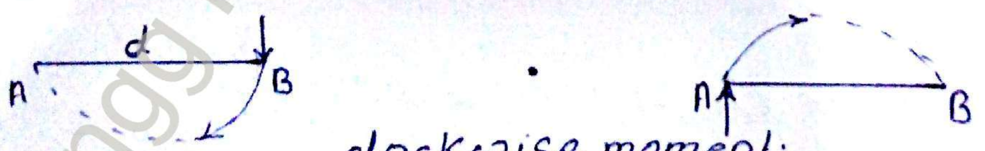
(i) It tends to move the body.

(ii) It tends to rotate the body.

⇒ unit of moment :- N·M or KN·M

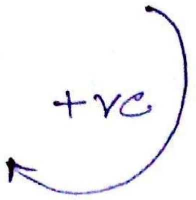
Direction of moment:-

The turning/rotational effect due to force can be clockwise or anticlockwise.

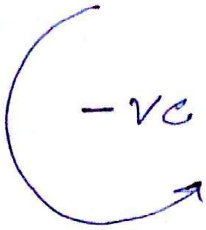


clockwise moment.

Anticlockwise moment



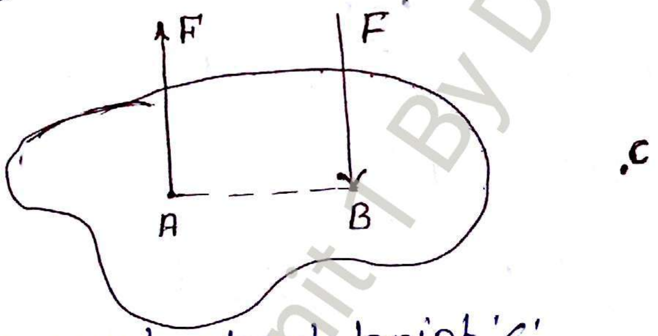
clockwise



Anticlockwise.

Couples:-

A special case of moment is a "couple". A couple consist of two parallel forces that are equal in magnitude & opposite in direction and do not have same line of action. It do not produce ~~only~~ any translation. It produce rotation only. The resultent force of a couple is zero but resultent of couple is not zero.



taking moment about point 'c'

$$M_c = F \times AC - F \times BC$$

$$= F(AC - BC)$$

$$= F(AB) \quad (\text{clockwise})$$

EX:-

→ The force exerted by your hand on a screw driver.

→ opening or closing a water tap.

→ Turn of the cap of a pen.

Characteristics of a couple:-

→ A couple consist of a pair of equal and opposite parallel force which are separated by definite distance.

→ The sum of two forces along any direction is zero. But the sum of moment about any given point is not zero.

→ The couples does not translate the body but it rotate the body.

→ Moment of a couple about any point is equal to the product of the force and perpendicular distance between the two forces.

$$\boxed{M = F \cdot d}$$

Varignon's theorem : Law of moment:-

" Moment of a resultant of two forces, about a point lying in a plane of the forces, is equal to the algebraic sum of moments of these two forces about the same point.

Equilibrium :-

Any system of forces (two or more than two forces) which keeps the body at rest is said to be in equilibrium

It means when the body is in equilibrium.

(i) The algebric sum of the component of the forces along the mutually perpendicular direction is zero.

(ii) The algebric sum of the component of the moment along each of the mutually perpendicular direction is zero.

i.e. $\sum F_x = 0$ & $\sum M_x = 0$

$\sum F_y = 0$ $\quad \sum M_y = 0$

$\sum F_z = 0$ $\quad \sum M_z = 0$

These are "Condition of equilibrium" for three mutually perpendicular axis.

→ In the case of coplaner forces (acting on x-y plane)

$\sum F_x = 0$ & $\sum M_z = 0$

$\sum F_y = 0$

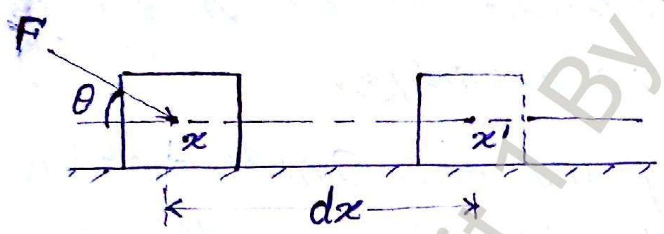
Virtual Work

Introduction :-

Work done is the dot product of the force and the displacement in the direction of force.

Let 'F' force is acting on a body and the body displaced by 'dx' distance than the work don by the force is.

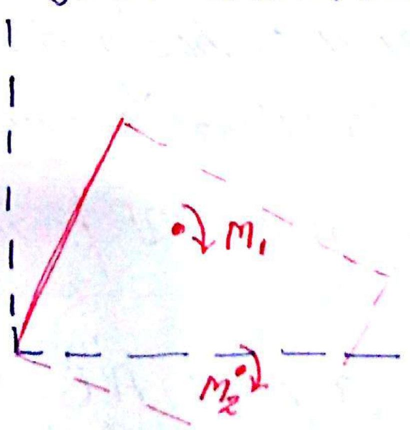
$$\boxed{Work = F \cdot dx = F dx \cos\theta}$$



Work done by a moment (couple)

A Couple 'M' acting on a body that change its angular position by an amount 'dθ' Then work done by the couple is.

$$\boxed{W = M \cdot d\theta}$$



work done by moment M_1

$= M_1 d\theta$

Work done by moment M_2

$= M_2 d\theta$

Virtual displacement and virtual work.

Consider a system of concurrent system F_1, F_2, F_3, ---- F_n acting on a particle.

Resultent forces 'R' is

$$R = \sum (F_1 + F_2 + F_3 --- + F_n)$$

If the system is in equilibrium than

$$R = \sum F = 0$$

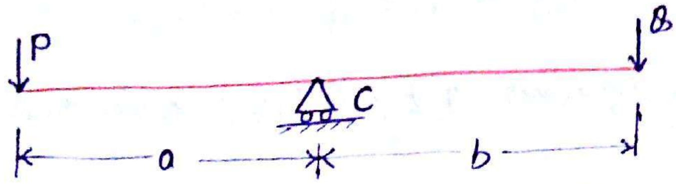
That means when the body is in equilibrium then the displacement of the body is zero and no work is posible.

But an imaginary infinite small displacement can be asumed to be given to the body in equilibrium. Such displacement is called "Virtual Displacement"

The resulting workdone by the force acting on the body during the virtual displacement is called "virtual work.

⇒ Application of principle of virtual work:-

Consider a bar ABC having support at 'c' and A & B points are free and P & Q forces acting on the point A & B.



Obtain the relation b/w P. and Q.

(i) Equilibrium eqn.

(ii) Principle of virtual work.

Soln

1. By Equilibrium

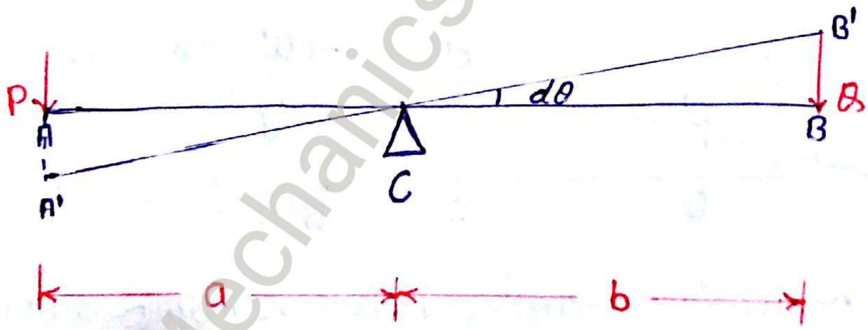
$$\sum M_c = 0$$

$$\Rightarrow \quad P \times a - Q \times b = 0$$

$$\boxed{P = \frac{b}{a} Q}$$

(ii) By principle of virtual work.

Let us consider a small angular displacement $d\theta$ in the given rod.



Displacement at A & B is

$$AA' = a \cdot d\theta \quad , \quad BB' = b d\theta$$

Now apply the principle of virtual work.

$$P(AA') = Q(BB')$$

$$P(a d\theta) = Q(b d\theta)$$

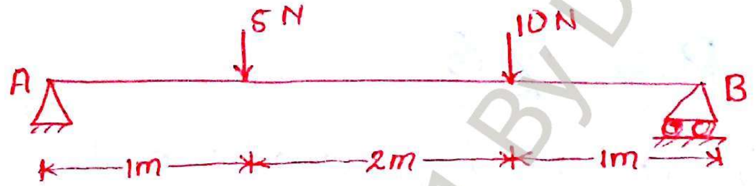
$$\boxed{P = \frac{b}{a} Q}$$

Sign Convention :-

1. Upward force taken a positive while downward is -ve.
2. Force acting towards right are positive and -ve.
3. Moment are positive if they are in clockwise direction.

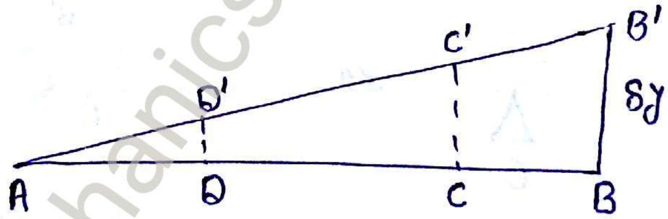
) +ve.

Q. Determine the reaction at A & B supports devloped in the beam in fig. by principle of virtual work.



Soln :-

Let The difflection at point 'B' is δy and displacement at A is zero. By virtual work ~~wart~~ principle.



From $\Delta ABB' \sim \Delta ACC'$ and From $\Delta ABB' \sim \Delta ADD'$

$$\frac{AB}{AC} = \frac{\delta y}{CC'} \qquad\qquad \frac{AB}{AD} = \frac{\delta y}{DD'}$$

$$\Rightarrow CC' = \frac{AC}{AB}\delta y \qquad\qquad \Rightarrow DD' = \frac{AD}{AB}\delta y.$$

$$\Rightarrow CC' = \frac{3}{4}\delta y \qquad\qquad \Rightarrow DD' = \frac{1}{4}\delta y.$$

From virtual work principle.

$$R_A \times 0 - 5 \times \frac{1}{4}\delta y - 10 \cdot \frac{3}{4}\delta y + R_A \cdot \delta y = 0$$

$$\Rightarrow \left(-\frac{5}{4} - \frac{30}{4} + R_B\right)\delta y = 0$$

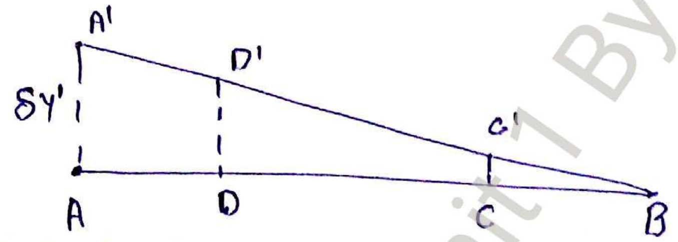
$$\delta y \neq 0$$

$$\Rightarrow -\frac{5}{4} - \frac{30}{4} + R_B = 0$$

$$R_B = \frac{5}{4} + \frac{30}{4} = \frac{35}{4}$$

$$\boxed{R_B = 8.75 \text{ KN}}$$

If $\delta y'$ displaced at support A and B is zero



from $\triangle BAA' \sim \triangle BDD'$

$$\frac{AB}{BD} = \frac{AA'}{DD'}$$

$$\Rightarrow DD' = \frac{BD}{AB}\delta y'$$

$$\Rightarrow \boxed{DD' = \frac{3}{4}\delta y'}$$

from $\triangle BAA' \sim \triangle BCC'$

$$\frac{AB}{BC} = \frac{AA'}{CC'}$$

$$\Rightarrow \frac{AB}{BC} = \frac{AA'}{CC'}$$

$$\Rightarrow CC' = \frac{BC}{AB} \cdot AA'$$

$$\boxed{CC' = \frac{1}{4}\delta y'}$$

By application of virtual work.

$$R_B \times 0 - \frac{1}{4}\delta y' \times 10 - \frac{3}{4}\delta y' \times 5 + R_A \cdot \delta y' = 0$$

$$\Rightarrow \delta y'\left(-\frac{10}{4} - \frac{15}{4} + R_A\right) = 0 \quad \text{Hear } \delta y' \neq 0$$

$$\left(-\frac{10}{4} - \frac{15}{4} + R_A\right) = 0$$

$$R_A = \frac{10}{4} + \frac{15}{4} = \frac{25}{4}$$

$$\boxed{R_A = 6.25 \text{ KN}}$$

$$\boxed{\begin{aligned} R_A &= 6.25 \text{ KN} \\ R_B &= 8.75 \text{ KN} \end{aligned}}$$